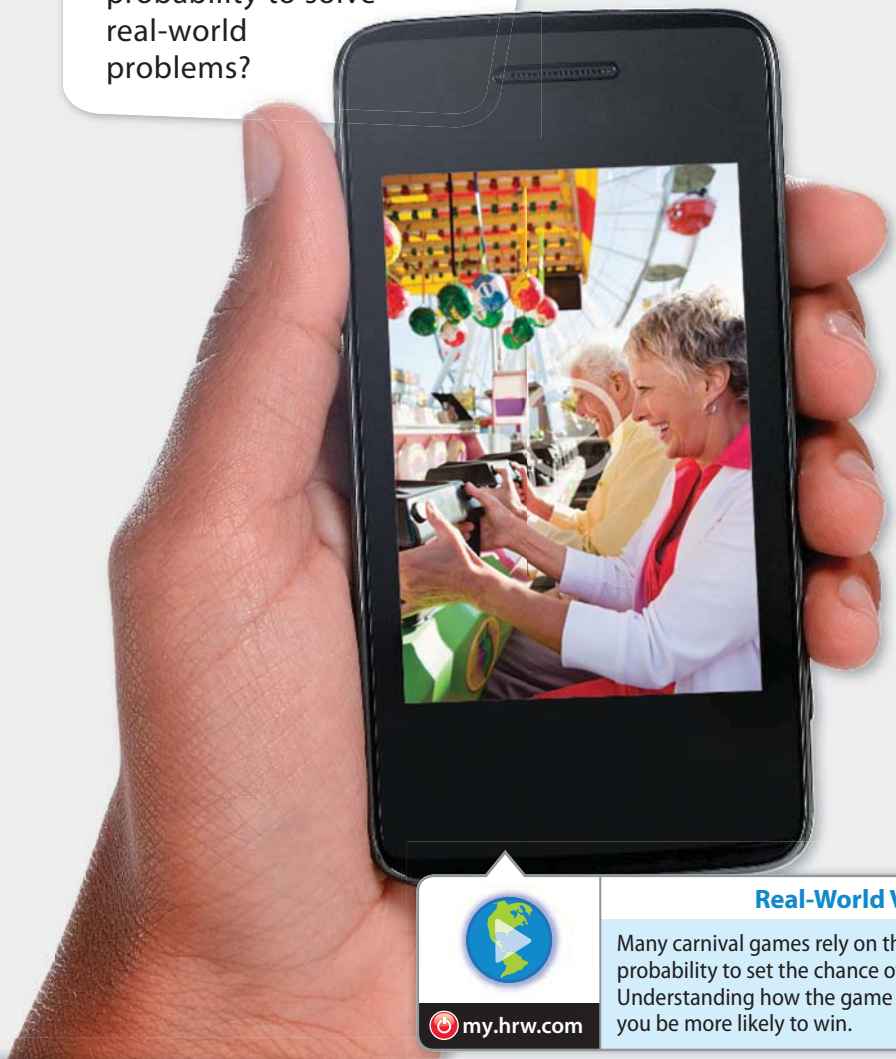
Theoretical Probability and Simulations

MODULE 13

COMMON CORE

ESSENTIAL QUESTION

How can you use theoretical probability to solve real-world problems?



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Real-World Video

Many carnival games rely on theoretical probability to set the chance of winning fairly low. Understanding how the game is set up might help you be more likely to win.

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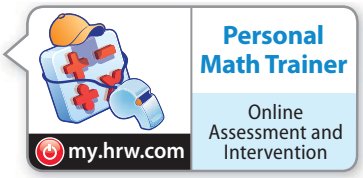
Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Fractions, Decimals, and Percents

EXAMPLE Write $\frac{3}{8}$ as a decimal and a percent.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Write the fraction as a division problem.
Write a decimal point and zeros in the dividend.
Place a decimal point in the quotient.
Divide as with whole numbers.

$0.375 = 37.5\%$. Write the decimal as a percent.

Write each fraction as a decimal and a percent.

1. $\frac{3}{4}$ _____ **2.** $\frac{2}{5}$ _____ **3.** $\frac{9}{10}$ _____ **4.** $\frac{7}{20}$ _____

5. $\frac{7}{8}$ _____ **6.** $\frac{1}{20}$ _____ **7.** $\frac{19}{25}$ _____ **8.** $\frac{23}{50}$ _____

Operations with Fractions

EXAMPLE $1 - \frac{7}{12} = \frac{12}{12} - \frac{7}{12}$ Use the denominator of the fraction to write 1 as a fraction.

$= \frac{12 - 7}{12}$ Subtract the numerators.

$= \frac{5}{12}$ Simplify.

Find each difference.

9. $1 - \frac{1}{5}$ _____ **10.** $1 - \frac{2}{9}$ _____ **11.** $1 - \frac{8}{13}$ _____ **12.** $1 - \frac{3}{20}$ _____

Multiply Fractions

EXAMPLE $\frac{4}{15} \times \frac{5}{6} = \frac{{}^{2}\cancel{4}}{\cancel{15}_{3}} \times \frac{{}^{1}\cancel{5}}{\cancel{6}_{3}}$ Divide by the common factors.

$= \frac{2}{9}$ Simplify.

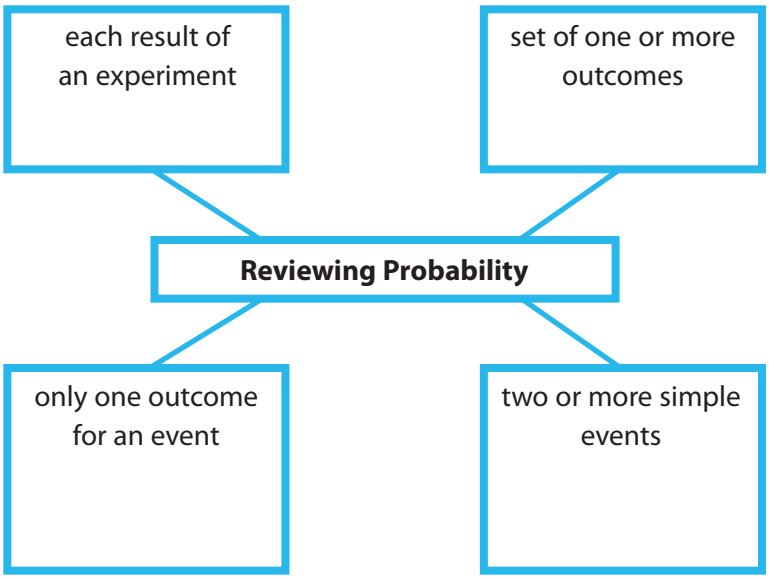
Multiply. Write each product in simplest form.

13. $\frac{8}{15} \times \frac{5}{8}$ _____ **14.** $\frac{2}{9} \times \frac{3}{4}$ _____ **15.** $\frac{9}{16} \times \frac{12}{13}$ _____ **16.** $\frac{7}{10} \times \frac{5}{28}$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.



Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. compound event
2. theoretical probability
3. complement

A. The set of all outcomes that are not the desired event.

B. An event made of two or more simple events.

C. The ratio of the number of equally likely outcomes in an event to the total number of possible outcomes.

Vocabulary

Review Words

complement *(complemento)*

✔ compound event *(suceso compuesto)*

✔ event *(suceso)*

experiment *(experimento)*

✔ outcome *(resultado)*

✔ simple event *(suceso simple)*

probability *(probabilidad)*

Preview Words

theoretical probability *(probabilidad teórica)*

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart, to help you understand the concepts in this module. Label one flap "Simple Events" and the other flap "Compound Events." As you study each lesson, write important ideas under the appropriate flap. Include information that will help you remember the concepts later when you look back at your notes.

MODULE 13

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **7.SP.7a**

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

What It Means to You

You will find the probabilities of a simple event and its complement.

UNPACKING EXAMPLE 7.SP.7A

Tara has a bag that contains 8 white marbles, 10 green marbles, and 7 red marbles. She selects a marble at random. Find the probability that the marble is red, and the probability that it is **not** red.

$$P(\text{red}) = \frac{\text{number of red marbles}}{\text{total number of marbles}}$$

$$= \frac{7}{25}$$

$$P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{7}{25} = \frac{25}{25} - \frac{7}{25} = \frac{18}{25}$$

The probability that the marble is red is $\frac{7}{25}$, and the probability that it is not red is $\frac{18}{25}$.

COMMON CORE **7.SP.8b**

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Key Vocabulary

compound event *(suceso compuesto)*
An event made of two or more simple events.

What It Means to You

You will identify the outcomes in the sample space of a compound event.

UNPACKING EXAMPLE 7.SP.8B

Identify the sample space for flipping a coin and rolling a number cube.

Make a table to organize the information.

		Number Cube Outcomes					
		1	2	3	4	5	6
COIN	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

The sample space includes 12 possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, and T6.

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LESSON 13.1

Theoretical Probability of Simple Events

COMMON CORE 7.SP.7a

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *Also 7.SP.6, 7.SP.7*

ESSENTIAL QUESTION

How can you find the theoretical probability of a simple event?

EXPLORE ACTIVITY 1 Real World

COMMON CORE 7.SP.7a

Finding Theoretical Probability

In previous lessons, you found probabilities based on observing data, or experimental probabilities. In this lesson, you will find *theoretical probabilities*.

At a school fair, you have a choice of spinning Spinner A or Spinner B. You win an MP3 player if the spinner lands on a section with a star in it. Which spinner should you choose if you want a better chance of winning?

Spinner A

Spinner B

A Complete the table.

	Spinner A	Spinner B
Total number of outcomes		
Number of sections with stars		
***P*(winning MP3)** $= \frac{\text{number of sections with stars}}{\text{total number of outcomes}}$		

B Compare the ratios for Spinner A and Spinner B.

The ratio for Spinner _____ is greater than the ratio for Spinner _____.

I should choose _____ for a better chance of winning.

Math Talk
Mathematical Practices

Describe a way to change Spinner B to make your chances of winning equal to your chances of not winning. Explain.

Reflect

1. *Theoretical probability* is a way to describe how you found the chance of winning an MP3 player in the scenario above. Using the spinner example to help you, explain in your own words how to find the theoretical probability of an event.

_____

_____

_____

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Calculating Theoretical Probability of Simple Events

Theoretical probability is the probability that an event occurs when all of the outcomes of the experiment are equally likely.

Theoretical Probability

$$P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

Probability can be written as a fraction, a decimal, or a percent. For example, the probability you win with Spinner B is $\frac{5}{16}$. You can also write that as 0.3125 or as 31.25%.

EXAMPLE 1

Real World

COMMON CORE 7.SP.7a

A bag contains 6 red marbles and 12 blue ones. You select one marble at random from the bag. What is the probability that you select a red marble? Write your answer in simplest form.

STEP 1 Find the number of ways the event can occur, that is, the number of red marbles: 6

STEP 2 Add to find the total number of equally likely outcomes.

number of red marbles	+	number of blue marbles	=	total number of marbles
6	+	12	=	**18**

There are 18 possible outcomes in the sample space.

STEP 3 Find the probability of selecting a red marble.

$$P(\text{red marble}) = \frac{\text{number of red marbles}}{\text{total number of marbles}} = \frac{6}{\mathbf{18}}$$

The probability that you select a red marble is $\frac{6}{18}$, or $\frac{1}{3}$.

Math Talk
Mathematical Practices
Describe a situation that has a theoretical probability of $\frac{1}{4}$.

YOUR TURN

2. You roll a number cube one time. What is the probability that you roll a 3 or 4? Write your answer in simplest form.

$P(\text{rolling a 3 or 4}) = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

3. How is the sample space for an event related to the formula for theoretical probability? _____

_____

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EXPLORE ACTIVITY 2 Real World

COMMON CORE 7.SP.7, 7.SP.6

Comparing Theoretical and Experimental Probability

Now that you have calculated theoretical probabilities, you may wonder how theoretical and experimental probabilities compare.

Six students are performing in a talent contest. You roll a number cube to determine the order of the performances.

STEP 1 You roll the number cube once. Complete the table of theoretical probabilities for the different outcomes.

Number	1	2	3	4	5	6
Theoretical probability						

STEP 2 Predict the number of times each number will be rolled out of 30 total rolls.

1: ☐ times 3: ☐ times 5: ☐ times

2: ☐ times 4: ☐ times 6: ☐ times

STEP 3 Roll a number cube 30 times. Complete the table for the frequency of each number and then find its experimental probability.

Number	1	2	3	4	5	6
Frequency						
Experimental probability						

STEP 4 Look at the tables you completed. How do the experimental probabilities compare with the theoretical probabilities?

_____

_____

STEP 5 **Conjecture** By performing more trials, you tend to get experimental results that are closer to the theoretical probabilities. Combine your table from **Step 3** with those of your classmates to make one table for the class. How do the class experimental probabilities compare with the theoretical probabilities?

_____

_____

EXPLORE ACTIVITY 2 *(cont'd)*

Reflect

4. Could the experimental probabilities ever be exactly equal to the theoretical probability? If so, how likely is it? If not, why not?

_____

_____

Guided Practice

At a school fair, you have a choice of randomly picking a ball from Basket A or Basket B. Basket A has 5 green balls, 3 red balls, and 8 yellow balls. Basket B has 7 green balls, 4 red balls, and 9 yellow balls. You can win a digital book reader if you pick a red ball. (Explore Activity 1)

	Basket A	Basket B
Total number of outcomes		
Number of red balls		
***P*(win) =** $\frac{\text{number of red balls}}{\text{total number of outcomes}}$		

1. Complete the chart. Write each answer in simplest form.

2. Which basket should you choose if you want the better chance of winning? _____

A spinner has 11 equal-sized sections marked 1 through 11. Find each probability. (Example 1)

3. You spin once and land on an odd number.

$P(\text{odd}) = \frac{\text{number of sections}}{\text{total number of}} = \frac{\square}{\square}$

4. You spin once and land on an even number.

$P(\text{even}) = \frac{\text{number of sections}}{\text{total number of}} = \frac{\square}{\square}$

You roll a number cube once.

5. What is the theoretical probability that you roll a 3 or 4? (Example 1) _____

6. Suppose you rolled the number cube 199 more times. Would you expect the experimental probability of rolling a 3 or 4 to be the same as your answer to Exercise 5? (Explore Activity 2)

_____

ESSENTIAL QUESTION CHECK-IN

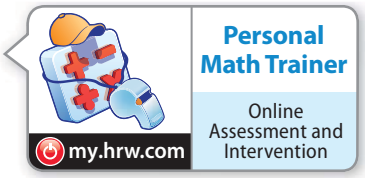
7. How can you find the probability of a simple event if the total number of equally likely outcomes is 20?

_____

Name _____ Class _____ Date _____

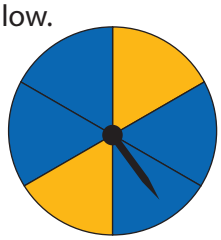
13.1 Independent Practice

COMMON CORE 7.SP.7, 7.SP.7a



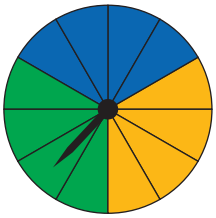
Find the probability of each event. Write each answer as a fraction in simplest form, as a decimal to the nearest hundredth, and as a percent to the nearest whole number.

8. You spin the spinner shown. The spinner lands on yellow.



_____

9. You spin the spinner shown. The spinner lands on blue or green.



_____

10. A jar contains 4 cherry cough drops and 10 honey cough drops. You choose one cough drop without looking. The cough drop is cherry. _____

11. You pick one card at random from a standard deck of 52 playing cards. You pick a black card. _____

12. There are 12 pieces of fruit in a bowl. Five are lemons and the rest are limes. You choose a piece of fruit without looking. The piece of fruit is a lime. _____

13. You choose a movie disc at random from a case containing 8 comedy discs, 5 science fiction discs, and 7 adventure discs. The disc is **not** a comedy. _____

14. You roll a number cube. You roll a number that is greater than 2 and less than 5. _____

15. **Communicate Mathematical Ideas** The theoretical probability of a given event is $\frac{9}{13}$. Explain what each number represents.

_____

_____

_____

16. Leona has 4 nickels, 6 pennies, 4 dimes, and 2 quarters in a change purse. Leona lets her little sister Daisy pick a coin at random. If Daisy is equally likely to pick each type of coin, what is the probability that her coin is worth more than five cents? Explain.

_____

_____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

17. **Critique Reasoning** A bowl of flower seeds contains 5 petunia seeds and 15 begonia seeds. Riley calculated the probability that a randomly selected seed is a petunia seed as $\frac{1}{3}$. Describe and correct Riley's error.

18. There are 20 seventh graders and 15 eighth graders in a club. A club president will be chosen at random.

a. **Analyze Relationships** Compare the probabilities of choosing a seventh grader or an eighth grader.

b. **Critical Thinking** If a student from one grade is more likely to be chosen than a student from the other, is the method unfair? Explain.

A jar contains 8 red marbles, 10 blue ones, and 2 yellow ones. One marble is chosen at random. The color is recorded in the table, and then it is returned to the jar. This is repeated 40 times.

Red	Blue	Yellow
14	16	10

19. **Communicate Mathematical Ideas** Use proportional reasoning to explain how you know that for each color, the theoretical and experimental probabilities are not the same.

20. **Persevere in Problem Solving** For which color marble is the experimental probability closest to the theoretical probability? Explain.

LESSON 13.2

Theoretical Probability of Compound Events

COMMON CORE 7.SP.8

Find probabilities of compound events using organized lists, tables, tree diagrams, ... *7.SP.8a, 7.SP.8b*

ESSENTIAL QUESTION

How do you find the probability of a compound event?

EXPLORE ACTIVITY COMMON CORE 7.SP.8, 7.SP.8a, 7.SP.8b

Finding Probability Using a Table

Recall that a compound event consists of two or more simple events. To find the probability of a compound event, you write a ratio of the number of ways the compound event can happen to the total number of equally likely possible outcomes.

Jacob rolls two fair number cubes. Find the probability that the sum of the numbers he rolls is 8.

STEP 1 Use the table to find the sample space for rolling a particular sum on two number cubes. Each cell is the sum of the first number in that row and column.

STEP 2 How many possible outcomes are in the sample space? _____

STEP 3 Circle the outcomes that give the sum of 8.

STEP 4 How many ways are there to roll a sum of 8? _____

STEP 5 What is the probability of rolling a sum of 8? _____

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Reflect

1. Give an example of an event that is more likely than rolling a sum of 8.

_____

2. Give an example of an event that is less likely than rolling a sum of 8.

_____

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Finding Probability Using a Tree Diagram

You can also use a tree diagram to calculate theoretical probabilities of compound events.

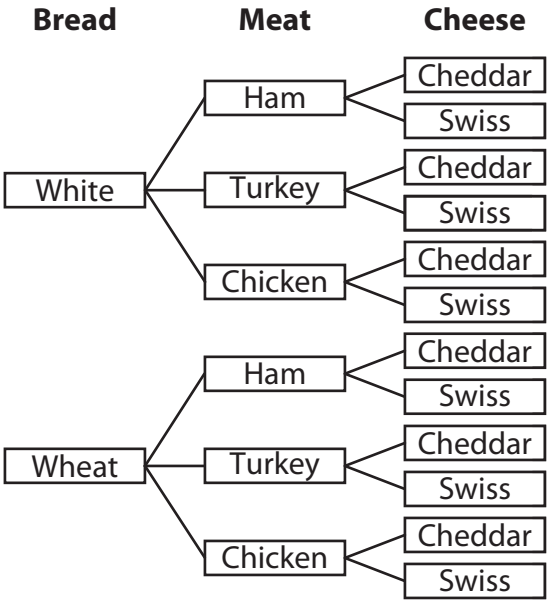
EXAMPLE 1

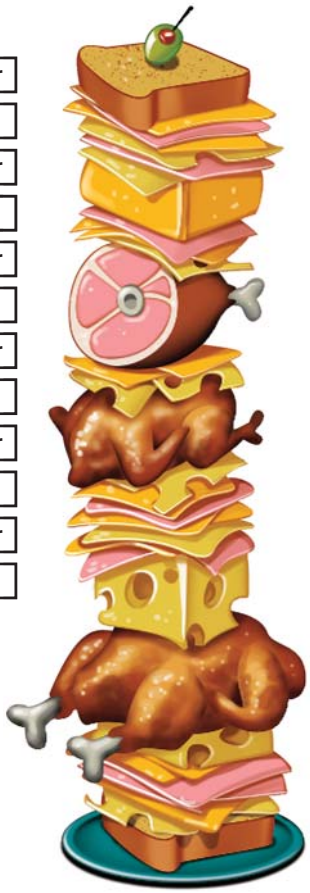
Real World

COMMON CORE 7.SP.8, 7.SP.8b

A deli prepares sandwiches with one type of bread (white or wheat), one type of meat (ham, turkey, or chicken), and one type of cheese (cheddar or Swiss). Each combination is equally likely. Find the probability of choosing a sandwich at random and getting turkey and Swiss on wheat bread.

STEP 1 Make a tree diagram to find the sample space for the compound event.





Math Talk
Mathematical Practices
How many sandwich combinations are possible if one of the meat options is unavailable?

STEP 2 Find the number of possible outcomes in the sample space: **12**

STEP 3 Find the probability of choosing turkey and Swiss on wheat bread at random: $\frac{1}{12}$

YOUR TURN

Use the diagram from Example 1 to find the given probabilities.

3. ham sandwich _____

4. sandwich containing Swiss cheese _____

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Finding Probability Using a List

One way to provide security for a locker or personal account is to assign it an access code number known only to the owner.

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EXAMPLE 2

Real World

COMMON CORE 7.SP.8, 7.SP.8b

The combination for Khiem's locker is a 3-digit code that uses the numbers 1, 2, and 3. Any of these numbers may be repeated. Find the probability that Khiem's randomly-assigned number is 222.

Make an organized list to find the sample space.

STEP 1 List all the codes that start with 1 and have 1 as a second digit.

1	1	1
1	1	2
1	1	3

STEP 2 List all the codes that start with 1 and have 2 as a second digit.

1	2	1
1	2	2
1	2	3

STEP 3 List all the codes that start with 1 and have 3 as a second digit.

1	3	1
1	3	2
1	3	3

STEP 4 You have now listed all the codes that start with 1. Repeat Steps 1–3 for codes that start with 2, and then for codes that start with 3.

2	1	1
2	1	2
2	1	3

2	2	1
2	2	2
2	2	3

2	3	1
2	3	2
2	3	3

3	1	1
3	1	2
3	1	3

3	2	1
3	2	2
3	2	3

3	3	1
3	3	2
3	3	3

Notice that there are 3 possible first numbers, 3 possible second numbers, and 3 possible third numbers, or $3 \times 3 \times 3 = 27$ numbers in all.

STEP 5 Find the number of outcomes in the sample space by counting all the possible codes. There are **27** such codes.

STEP 6 Find the probability that Khiem's locker code is 222.

$$P(\text{Code 222}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \frac{1}{27}$$

Math Talk
Mathematical Practices

How could you find the probability that Khiem's locker code includes exactly two 1s?

YOUR TURN

5. Martha types a 4-digit code into a keypad to unlock her car doors. The code uses the numbers 1 and 0. If the digits are selected at random, what is the probability of getting a code with exactly two 0s? _____

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My Notes

Guided Practice

Drake rolls two fair number cubes. (Explore Activity)

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

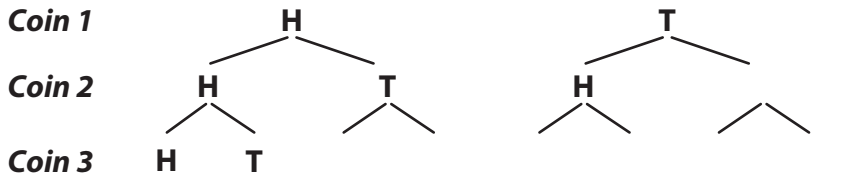
1. Complete the table to find the sample space for rolling a particular product on two number cubes.

2. What is the probability that the product of the two numbers Drake rolls is a multiple of 4?_____

3. What is the probability that the product of the two numbers Drake rolls is less than 13? _____

You flip three coins and want to explore probabilities of certain events. (Examples 1 and 2)

4. Complete the tree diagram and make a list to find the sample space.



List: HHH HHT _____ _____ T___ T___ _____ _____

5. How many outcomes are in the sample space? _____

6. List all the ways to get three tails. _____

7. Complete the expression to find the probability of getting three tails.

$$P = \frac{\text{number of outcomes with } \square}{\text{total number of possible outcomes}} = \frac{\square}{\square}$$

The probability of getting three tails when three coins are flipped is _____.

8. What is the probability of getting exactly two heads?

There are _____ way(s) to obtain exactly two heads: HHT, _____

$$P = \frac{\text{number of outcomes with } \square}{\text{total number of possible outcomes}} = \frac{\square}{\square}$$

ESSENTIAL QUESTION CHECK-IN

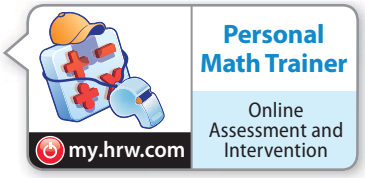
9. There are 6 ways a given compound event can occur. What else do you need to know to find the theoretical probability of the event?

_____

Name _____ Class _____ Date _____

13.2 Independent Practice

COMMON CORE 7.SP.8, 7.SP.8a, 7.SP.8b



In Exercises 10–12, use the following information. Mattias gets dressed in the dark one morning and chooses his clothes at random. He chooses a shirt (green, red, or yellow), a pair of pants (black or blue), and a pair of shoes (checkered or red).

10. Use the space below to make a tree diagram to find the sample space.

11. What is the probability that Mattias picks an outfit at random that includes red shoes? _____

12. What is the probability that no part of Mattias's outfit is red? _____

13. Rhee and Pamela are two of the five members of a band. Every week, the band picks two members at random to play on their own for five minutes. What is the probability that Rhee and Pamela are chosen this week? _____

14. Ben rolls two number cubes. What is the probability that the sum of the numbers he rolls is less than 6? _____

15. Nhan is getting dressed. He considers two different shirts, three pairs of pants, and three pairs of shoes. He chooses one of each of the articles at random. What is the probability that he will wear his jeans but not his sneakers?

Shirt	Pants	Shoes
collared	khakis	sneakers
T-shirt	jeans	flip-flops
	shorts	sandals

_____

16. **Communicate Mathematical Ideas** A ski resort has 3 chair lifts, each with access to 6 ski trails. Explain how you can find the number of possible outcomes when choosing a chair lift and a ski trail without making a list, a tree diagram, or table.

_____

_____

_____

_____

17. **Explain the Error** For breakfast, Sarah can choose eggs, granola or oatmeal as a main course, and orange juice or milk for a drink. Sarah says that the sample space for choosing one of each contains $3^2 = 9$ outcomes. What is her error? Explain.

_____

_____

_____

18. **Represent Real-World Problems** A new shoe comes in two colors, black or red, and in sizes from 5 to 12, including half sizes. If a pair of the shoes is chosen at random for a store display, what is the probability it will be

red and size 9 or larger? _____

H.O.T. FOCUS ON HIGHER ORDER THINKING

19. **Analyze Relationships** At a diner, Sondra tells the server, "Give me one item from each column." Gretchen says, "Give me one main dish and a vegetable." Who has a greater probability of getting a meal that includes salmon? Explain.

Main Dish	Vegetable	Side
Pasta	Carrots	Tomato soup
Salmon	Peas	Tossed salad
Beef	Asparagus	
Pork	Sweet potato	

_____

_____

20. The digits 1 through 5 are used for a set of locker codes.

a. **Look for a Pattern** Suppose the digits cannot repeat. Find the number of possible two-digit codes and three-digit codes. Describe any pattern and use it to predict the number of possible five-digit codes.

_____

_____

_____

b. **Look for a Pattern** Repeat part **a**, but allow digits to repeat.

_____

_____

_____

c. **Justify Reasoning** Suppose that a gym plans to issue numbered locker codes by choosing the digits at random. Should the gym use codes in which the digits can repeat or not? Justify your reasoning.

_____

_____

_____

Work Area

LESSON
13.3 Making Predictions with Theoretical Probability

COMMON CORE 7.SP.6

. . . predict the approximate relative frequency given the probability. *Also 7.RP.3, 7.SP.7a*

ESSENTIAL QUESTION

How do you make predictions using theoretical probability?

EXPLORE ACTIVITY Real World

COMMON CORE 7.SP.6

Using Theoretical Probability to Make a Quantitative Prediction

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You can make quantitative predictions based on theoretical probability just as you did with experimental probability earlier.

EXAMPLE 1 **Use proportional reasoning to solve each problem.**

A **You roll a standard number cube 150 times. Predict how many times you will roll a 3 or a 4.**

The probability of rolling a 3 or a 4 is $\frac{2}{6} = \frac{1}{3}$.

Method 1: Set up a proportion.

Write a proportion. The ratio 1 out of 3 is how many out of 150?

$$\frac{\square}{3} = \frac{\square}{150}$$

Since 3 times _____ is 150, multiply 1 by _____to find the value of x.

$$\frac{\square \times}{3 \times} = \frac{\square}{150}$$

$x =$ _____

Method 2: Set up an equation and solve.

p(rolling a 3 or 4) · Number of events = Prediction

Multiply the probability by the total number of rolls.

$$\frac{\square}{\square} \cdot \square = x$$

Solve for x.

_____ $= x$

You can expect to roll a 3 or a 4 about _____ times out of 150.

EXPLORE ACTIVITY *(cont'd)*

B **Celia volunteers at her local animal shelter. She has an equally likely chance to be assigned to the dog, cat, bird, or reptile section. If she volunteers 24 times, about how many times should she expect to be assigned to the dog section?**

The probability of being assigned to the dog section is $\frac{\square}{\square}$.

Write a proportion. The ratio 1 out of 4 is how many out of 24?

$$\frac{\square}{4} = \frac{\square}{24}$$

Since 4 times ____ is 24, multiply 1 by ____ to find the value of x.

$$\frac{\square}{4} \times \square = \frac{\square}{24} \times \square$$

Celia can expect to be assigned to the dog section about _____ times out of 24.

YOUR TURN

1. Predict how many times you will roll a number less than 5 if you roll a standard number cube 250 times.

_____

2. You flip a fair coin 18 times. About how many times would you expect heads to appear?

_____

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Using Theoretical Probability to Make a Qualitative Prediction

Earlier, you learned how to make predictions using experimental probability. You can use theoretical probabilities in the same way to help you predict or compare how likely events are.

EXAMPLE 2

Real World

COMMON CORE 7.SP.6, 7.SP.7a

A **Herschel pulls a sock out of his drawer without looking and puts it on. The sock is black. There are 7 black socks, 8 white socks, and 5 striped socks left in the drawer. He pulls out a second sock without looking. Is it likely that he will be wearing matching socks to school?**

Find the theoretical probability that Herschel picks a matching sock and the probability that he picks one that does not match.

$P(\text{matching}) = \frac{7}{20}$ $P(\text{not matching}) = 1 - \frac{7}{20} = \frac{13}{20}$

P(not matching) = 1 − P(matching)

The probability that Herschel picks a matching sock is about half the probability that he picks one that does not match. It is likely that he will **not** be wearing matching socks to school.

B **All 2,000 customers at a gym are randomly assigned a 3-digit security code that they use to access their online accounts. The codes are made up of the digits 0 through 4, and the digits can be repeated. Is it likely that fewer than 10 of the customers are issued the code 103?**

Set up a proportion. The probability of the code 103 is $\frac{1}{125}$.

$\frac{1}{125} = \frac{x}{2{,}000}$ *Write a proportion. 1 out of 125 is how many out of 2,000?*

$\frac{1}{125} \overset{\times 16}{=} \frac{16}{2{,}000}$ *Since 125 times 16 is 2,000, multiply 1 times 16 to find the value of x.*

There are 5 possible first numbers, 5 possible second numbers, and 5 possible third numbers. So, the probability of any one code is $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125}$.

It is **not** likely that fewer than 10 of the customers get the same code. It is more likely that 16 members get the code 103.

YOUR TURN

3. A bag of marbles contains 8 red marbles, 4 blue marbles, and 5 white marbles. Tom picks a marble at random. Is it more likely that he picks a red marble or a marble of another color?

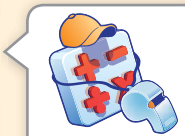
_____

4. At a fundraiser, a school group charges $6 for tickets for a "grab bag." You choose one bill at random from a bag that contains 40 $1 bills, 20 $5 bills, 5 $10 bills, 5 $20 bills, and 1 $100 bill. Is it likely that you will win enough to pay for your ticket? Justify your answer.

_____

_____

_____



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Guided Practice

1. Bob works at a construction company. He has an equally likely chance to be assigned to work different crews every day. He can be assigned to work on crews building apartments, condominiums, or houses. If he works 18 days a month, about how many times should he expect to be assigned to the house crew? (Explore Activity Example 1)

STEP 1 Find the probabilities of being assigned to each crew.

Apartment ☐ Condo ☐ House ☐

The probability of being assigned to the house crew is _____

STEP 2 Set up and solve a proportion.

$\frac{\square}{\square} = \frac{x}{\square}$ $x =$ _____

Bob can expect to be assigned to the house crew about _____ times out of 18.

2. During a raffle drawing, half of the ticket holders will receive a prize. The winners are equally likely to win one of three prizes: a book, a gift certificate to a restaurant, or a movie ticket. If there are 300 ticket holders, predict the number of people who will win a movie ticket.

(Explore Activity Example 1) _____

3. In Mr. Jawarani's first period math class, there are 9 students with hazel eyes, 10 students with brown eyes, 7 students with blue eyes, and 2 students with green eyes. Mr. Jawarani picks a student at random. Which color eyes is the student most likely to have? Explain. (Example 2)

_____

_____

ESSENTIAL QUESTION CHECK-IN

4. How do you make predictions using theoretical probability?

_____

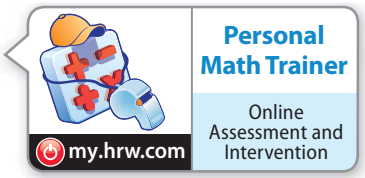
_____

_____

Name _____ Class _____ Date _____

13.3 Independent Practice

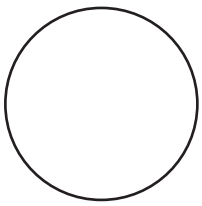
COMMON CORE 7.SP.6, 7.SP.7a



5. A bag contains 6 red marbles, 2 white marbles, and 1 gray marble. You randomly pick out a marble, record its color, and put it back in the bag. You repeat this process 45 times. How many white or gray marbles do you expect to get?

_____

6. Using the blank circle below, draw a spinner with 8 equal sections and 3 colors—red, green, and yellow. The spinner should be such that you are equally likely to land on green or yellow, but more likely to land on red than either on green or yellow.



Use the following for Exercises 7–9.
In a standard 52-card deck, half of the cards are red and half are black. The 52 cards are divided evenly into 4 suits: spades, hearts, diamonds, and clubs. Each suit has three face cards (jack, queen, king), and an ace. Each suit also has 9 cards numbered from 2 to 10.

7. Dawn draws 1 card, replaces it, and draws another card. Is it more likely that she draws 2 red cards or 2 face cards?

_____

_____

8. Luis draws 1 card from a deck, 39 times. Predict how many times he draws an ace.

_____

9. Suppose a solitaire player has played 1,000 games. Predict how many times the player turned over a red card as the first card.

_____

10. John and O'Neal are playing a board game in which they roll two number cubes. John needs to get a sum of 8 on the number cubes to win. O'Neal needs a sum of 11. If they take turns rolling the number cube, who is more likely to win? Explain.

_____

_____

_____

11. Every day, Navya's teacher randomly picks a number from 1 to 20 to be the number of the day. The number of the day can be repeated. There are 180 days in the school year. Predict how many days the number of the day will be greater than 15. _____

12. Eben rolls two standard number cubes 36 times. Predict how many times he will roll a sum of 4. _____

13. **Communicate Mathematical Ideas** Can you always show that a prediction based on theoretical probability is true by performing the event often enough? If so, explain why. If not, describe a situation that justifies your response.

_____

_____

_____

_____

_____

14. **Represent Real-World Problems** Give a real-world example of an experiment in which all of the outcomes are not equally likely. Can you make a prediction for this experiment, using theoretical probability?

H.O.T. FOCUS ON HIGHER ORDER THINKING

15. **Critical Thinking** Pierre asks Sherry a question involving the theoretical probability of a compound event in which you flip a coin and draw a marble from a bag of marbles. The bag of marbles contains 3 white marbles, 8 green marbles, and 9 black marbles. Sherry's answer, which is correct, is $\frac{12}{40}$. What was Pierre's question?

16. **Make a Prediction** Horace is going to roll a standard number cube and flip a coin. He wonders if it is more likely that he rolls a 5 **and** the coin lands on heads, or that he rolls a 5 **or** the coin lands on heads. Which event do you think is more likely to happen? Find the probability of both events to justify or reject your initial prediction.

17. **Communicate Mathematical Ideas** Cecil solved a theoretical prediction problem and got this answer: "The spinner will land on the red section 4.5 times." Is it possible to have a prediction that is not a whole number? If so, give an example.

Work Area

LESSON 13.4

Using Technology to Conduct a Simulation

COMMON CORE 7.SP.8c

Design and use a simulation to generate frequencies for compound events. *Also 7.SP.8*

ESSENTIAL QUESTION

How can you use technology simulations to estimate probabilities?

EXPLORE ACTIVITY Real World

COMMON CORE 7.SP.8c

Designing and Conducting a Simulation for a Simple Event

You can use a graphing calculator or computer to generate random numbers and conduct a simulation.

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EXAMPLE 1 **A cereal company is having a contest. There are codes for winning prizes in 30% of its cereal boxes. Find an experimental probability that you have to buy *exactly* 3 boxes of cereal before you find a winning code.**

STEP 1 Choose a model to simulate the event.

The probability of finding a winning code is 30% $= \frac{\square}{10}$.

Let 3 out of 10 numbers represent buying a box with a winning code. Use whole numbers from 1 to 10.

Winning: 1, 2, 3 Nonwinning: _____

STEP 2 Generate random numbers from 1 to 10 until you get one that represents a box with a winning code. Record how many boxes you bought before finding a winning code.

Sample Trial 1: 9, 6, 7, 8, 1

For Trial 1, you got the winning code ____ after buying ____ boxes.

Trial	Numbers generated	Boxes bought
1	9, 6, 7, 8, 1	5
2	2	1
3	10, 4, 8, 1	4
4	4, 10, 7, 1	4
5	2	1
6	4, 3	2
7	3	1
8	7, 5, 2	3
9	8, 5, 4, 8, 10, 3	6
10	9, 1	2

STEP 3 Perform multiple trials by repeating Step ____.

STEP 4 Find the experimental probability of the event.

Circle the trial(s) where you bought exactly ____ boxes of cereal before finding a winning code.
In ____ of 10 trials, you bought exactly 3 boxes of cereal before finding a winning code.

The experimental probability is $\frac{\square}{10}$ or ____ %.

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EXPLORE ACTIVITY *(cont'd)*

YOUR TURN

1. An elephant has a 50% chance of giving birth to a male or a female calf. Use a simulation to find an experimental probability that the elephant gives birth to 3 male calves before having a female calf. (*Hint:* Use 0s and 1s. Let 0 represent a male calf, and 1 represent a female calf. Generate random numbers until you get a 1.)

Trial	Numbers generated	3 Males first
1		
2		
3		
4		
5		

Trial	Numbers generated	3 Males first
6		
7		
8		
9		
10		

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Math Talk
Mathematical Practices
Could you generate random numbers from a list of more than 2 numbers? Explain.

Math On the Spot
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Designing and Conducting a Simulation for a Compound Event

You can use random numbers to simulate compound events as well as simple events.

EXAMPLE 2

Real World

COMMON CORE 7.SP.8c, 7.SP.3.8

Suppose that there is a 20% chance that a particular volcano will erupt in any given decade. Find an experimental probability that the volcano will erupt in at least 1 of the next 5 decades.

STEP 1 Choose a model.

The probability of an eruption is $20\% = \frac{1}{5}$.
Use whole numbers from 1 to 5.

Let 1 represent a decade with an eruption.

Let 2, 3, 4, and 5 represent a decade without an eruption.

STEP 2 Generate 5 random numbers from 1 to 5. Record the number of decades with an eruption.

5 numbers generated: 3, 1, 3, 4, 2 Eruption decades: 1

STEP 3 Perform multiple trials by repeating Step 2. Calculate the percent of trials in which there was an eruption in at least 1 of the 5 decades.

Trial	Numbers generated	Eruption decades
1	3, 1, 3, 4, 2	1
2	3, 2, 2, 4, 5	0
3	1, 3, 3, 2, 5	1
4	5, 3, 4, 5, 4	0
5	5, 5, 3, 2, 4	0

Trial	Numbers generated	Eruption decades
6	2, 3, 3, 4, 2	0
7	1, 2, 4, 1, 4	2
8	1, 3, 2, 1, 5	2
9	1, 2, 4, 2, 5	1
10	5, 5, 3, 2, 4	0

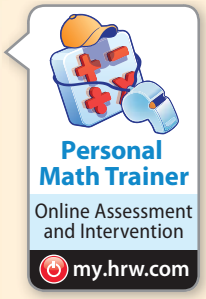
In 5 out of the 10 trials, there was an eruption in at least 1 of the 5 decades. The experimental probability of an eruption in at least 1 of the next 5 decades is $\frac{5}{10} = 50\%$.

YOUR TURN

2. Matt guesses the answers on a quiz with 5 true-false questions. The probability of guessing a correct answer on each question is 50%. Use a simulation to find an experimental probability that he gets at least 2 questions right. (*Hint:* Use 0s and 1s. Let 0s represent incorrect answers, and 1s represent correct answers. Perform 10 trials, generating 5 random numbers in each, and count the number of 1s.)

Trial	Numbers generated	Correct answers
1		
2		
3		
4		
5		

Trial	Numbers generated	Correct answers
6		
7		
8		
9		
10		



Guided Practice

There is a 30% chance that T'Shana's county will have a drought during any given year. She performs a simulation to find the experimental probability of a drought in at least 1 of the next 4 years.
(Explore Activity Example 1 and Example 2)

1. T'Shana's model involves the whole numbers from 1 to 10. Complete the description of her model.

Let the numbers 1 to 3 represent _____

and the numbers 4 to 10 represent _____

Perform multiple trials, generating _____ random numbers each time.

2. Suppose T'Shana used the model described in Exercise 1 and got the results shown in the table. Complete the table.

Trial	Numbers generated	Drought years
1	10, 3, 5, 1	
2	10, 4, 6, 5	
3	3, 2, 10, 3	
4	2, 10, 4, 4	
5	7, 3, 6, 3	

Trial	Numbers generated	Drought years
6	8, 4, 8, 5	
7	6, 2, 2, 8	
8	6, 5, 2, 4	
9	2, 2, 3, 2	
10	6, 3, 1, 5	

3. According to the simulation, what is the experimental probability that there will be a drought in the county in at least 1 of the next 4 years? _____

ESSENTIAL QUESTION CHECK-IN

4. You want to generate random numbers to simulate an event with a 75% chance of occurring. Describe a model you could use.

_____

_____

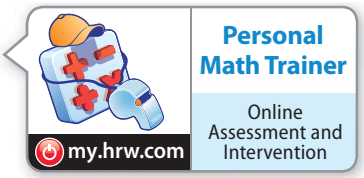
_____

_____

Name _____ Class _____ Date _____

13.4 Independent Practice

COMMON CORE 7.SP.8, 7.SP.8c



Every contestant on a game show has a 40% chance of winning. In the simulation below, the numbers 1–4 represent a winner, and the numbers 5–10 represent a nonwinner. Numbers were generated until one that represented a winner was produced.

Trial	Numbers generated
1	7, 4
2	6, 5, 2
3	1
4	9, 1
5	3

Trial	Numbers generated
6	8, 8, 6, 2
7	2
8	5, 9, 4
9	10, 3
10	1

5. In how many of the trials did it take exactly 4 contestants to get a winner? _____

6. Based on the simulation, what is the experimental probability that it will take exactly 4 contestants to get a winner? _____

Over a 100-year period, the probability that a hurricane struck Rob's city in any given year was 20%. Rob performed a simulation to find an experimental probability that a hurricane would strike the city in at least 4 of the next 10 years. In Rob's simulation, 1 represents a year with a hurricane.

Trial	Numbers generated
1	2, 5, 3, 2, 5, 5, 1, 4, 5, 2
2	1, 1, 5, 2, 2, 1, 3, 1, 1, 5
3	4, 5, 4, 5, 5, 4, 3, 5, 1, 1
4	1, 5, 5, 5, 1, 2, 2, 3, 5, 3
5	5, 1, 5, 3, 5, 3, 4, 5, 3, 2

Trial	Numbers generated
6	1, 1, 5, 5, 1, 4, 2, 2, 3, 4
7	2, 1, 5, 3, 1, 5, 1, 2, 1, 4
8	2, 4, 3, 2, 4, 4, 2, 1, 3, 1
9	3, 2, 1, 4, 5, 3, 5, 5, 1, 2
10	3, 4, 2, 4, 3, 5, 2, 3, 5, 1

7. According to Rob's simulation, what was the experimental probability that a hurricane would strike the city in at least 4 of the next 10 years? _____

8. **Analyze Relationships** Suppose that over the 10 years following Rob's simulation, there was actually 1 year in which a hurricane struck. How did this compare to the results of Rob's simulation?

_____

_____

9. **Communicate Mathematical Ideas** You generate three random whole numbers from 1 to 10. Do you think that it is unlikely or even impossible that all of the numbers could be 10? Explain?

10. Erika collects baseball cards, and 60% of the packs contain a player from her favorite team. Use a simulation to find an experimental probability that she has to buy exactly 2 packs before she gets a player from her favorite team.

H.O.T. FOCUS ON HIGHER ORDER THINKING

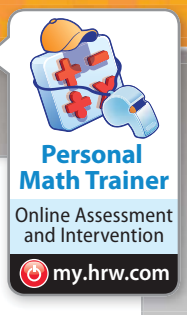
11. **Represent Real-World Problems** When Kate plays basketball, she usually makes 37.5% of her shots. Design and conduct a simulation to find the experimental probability that she makes at least 3 of her next 10 shots. Justify the model for your simulation.

12. **Justify Reasoning** George and Susannah used a simulation to simulate the flipping of 8 coins 50 times. In all of the trials, at least 5 heads came up. What can you say about their simulation? Explain.

Work Area

MODULE QUIZ

Ready to Go On?



13.1, 13.2 Theoretical Probability of Simple and Compound Events

Find the probability of each event. Write your answer as a fraction, as a decimal, and as a percent.

1. You choose a marble at random from a bag containing 12 red, 12 blue, 15 green, 9 yellow, and 12 black marbles. The marble is red. _____

2. You draw a card at random from a shuffled deck of 52 cards. The deck has four 13-card suits (diamonds, hearts, clubs, spades). The card is a diamond or a spade. _____

13.3 Making Predictions with Theoretical Probability

3. A bag contains 23 red marbles, 25 green marbles, and 18 blue marbles. You choose a marble at random from the bag. What color marble will you most likely choose? _____

13.4 Using Technology to Conduct a Simulation

4. Bay City has a 25% chance of having a flood in any given decade. The table shows the results of a simulation using random numbers to find the experimental probability that there will be a flood in Bay City in at least 1 of the next 5 decades. In the table, the number 1 represents a decade with a flood. The numbers 2 through 5 represent a decade without a flood.

Trial	Numbers generated	Trial	Numbers generated
1	2, 2, 5, 5, 5	6	4, 2, 2, 5, 4
2	3, 2, 3, 5, 4	7	1, 3, 2, 4, 4
3	5, 5, 5, 4, 3	8	3, 5, 5, 2, 1
4	5, 1, 3, 3, 5	9	4, 3, 3, 2, 5
5	4, 5, 5, 3, 2	10	5, 4, 1, 2, 1

According to the simulation, what is the experimental probability of a flood in Bay City in at least 1 of the next 5 decades? _____

ESSENTIAL QUESTION

5. How can you use theoretical probability to make predictions in real-world situations?

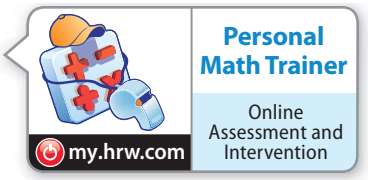
_____

_____

COMMON CORE

MODULE 13 MIXED REVIEW

Assessment Readiness



Selected Response

1. What is the probability of flipping two fair coins and having both show tails?

Ⓐ $\frac{1}{8}$ Ⓒ $\frac{1}{3}$

Ⓑ $\frac{1}{4}$ Ⓓ $\frac{1}{2}$

2. A bag contains 8 white marbles and 2 black marbles. You pick out a marble, record its color, and put the marble back in the bag. If you repeat this process 45 times, how many times would you expect to remove a white marble from the bag?

Ⓐ 9 Ⓒ 36

Ⓑ 32 Ⓓ 40

3. Philip rolls a standard number cube 24 times. Which is the best prediction for the number of times he will roll a number that is even and less than 4?

Ⓐ 2 Ⓒ 4

Ⓑ 3 Ⓓ 6

4. A set of cards includes 24 yellow cards, 18 green cards, and 18 blue cards. What is the probability that a card chosen at random is **not** green?

Ⓐ $\frac{3}{10}$ Ⓒ $\frac{3}{5}$

Ⓑ $\frac{4}{10}$ Ⓓ $\frac{7}{10}$

5. A rectangle made of square tiles measures 10 tiles long and 8 tiles wide. What is the width of a similar rectangle whose length is 15 tiles?

Ⓐ 3 tiles Ⓒ 13 tiles

Ⓑ 12 tiles Ⓓ 18.75 tiles

6. The Fernandez family drove 273 miles in 5.25 hours. How far would they have driven at that rate in 4 hours?

Ⓐ 208 miles Ⓒ 280 miles

Ⓑ 220 miles Ⓓ 358 miles

7. There are 20 tennis balls in a bag. Five are orange, 7 are white, 2 are yellow, and 6 are green. You choose one at random. Which color ball are you **least** likely to choose?

Ⓐ green Ⓒ white

Ⓑ orange Ⓓ yellow

Mini-Task

8. Center County has had a 1 in 6 (or about 16.7%) chance of a tornado in any given decade. In a simulation to consider the probability of tornadoes in the next 5 decades, Ava rolled a number cube. She let a 1 represent a decade with a tornado, and 2–6 represent decades without tornadoes. What experimental probability did Ava find for each event?

Trial	Numbers Generated	Trial	Numbers Generated
1	2, 2, 3, 1, 5	6	4, 5, 2, 2, 4
2	3, 5, 6, 4, 5	7	5, 1, 6, 3, 1
3	1, 3, 3, 2, 2	8	1, 2, 1, 2, 4
4	6, 3, 3, 5, 4	9	1, 4, 4, 1, 4
5	4, 1, 4, 4, 4	10	3, 6, 5, 3, 6

a. That Center County has a tornado in at least one of the next five decades.

_____

b. That Center County has a tornado in exactly one of the next five decades.

_____